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⑤ 3 dim'l hyperbolic manifolds as knot complements

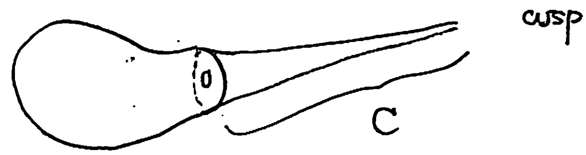
$K \subset S^3$ a hyperbolic knot

$M := S^3 \setminus K \cong \mathbb{H}^3/\Gamma$ $\qquad \Gamma \cong G(K)$

Γ : a Kleinian group $< \mathrm{Isom}^+ \mathbb{H}^3 \cong PSL(2,\mathbb{C})$.

$M = M_0 \cup C$

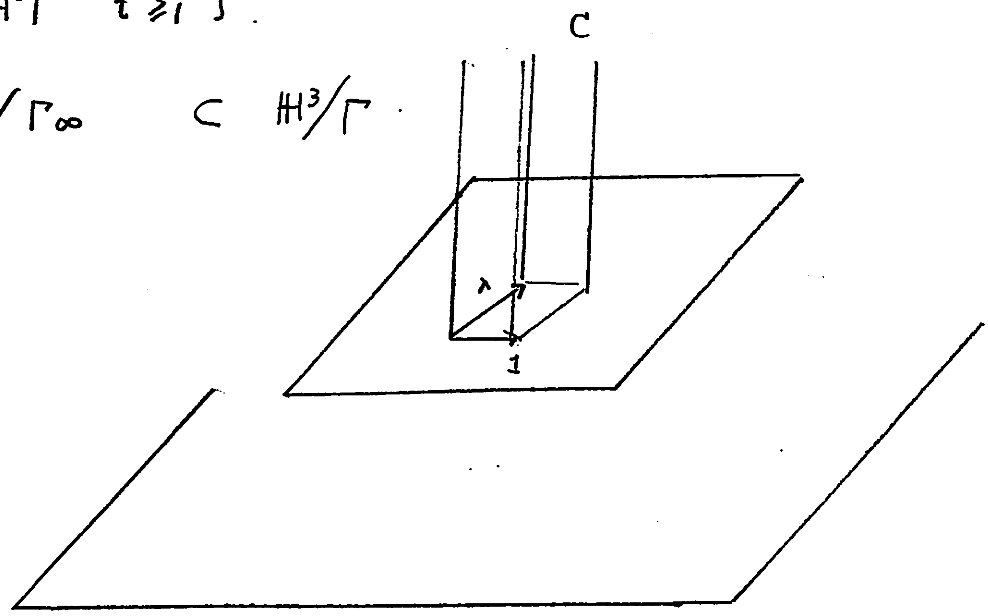
$M_0 \cong S^3 - \mathring{N}(K)$. $\qquad C \cong T \times [0,\infty)$



$\Gamma_\infty = \mathrm{Stab}(\infty, \Gamma) = \left\langle \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & \lambda \\ 0 & 1 \end{pmatrix} \right\rangle$

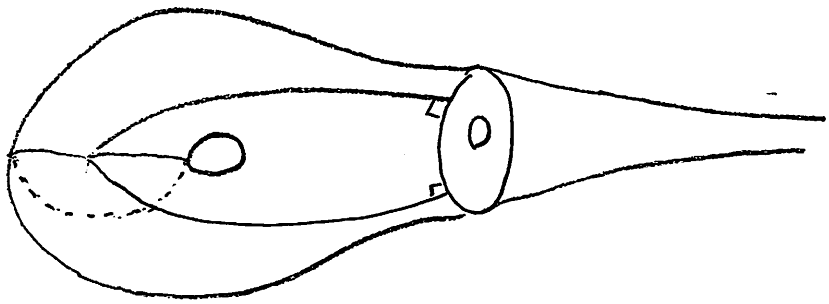
$\mathbb{H}_\infty = \{ (z,t) \in \mathbb{H}^3 \mid t \geq 1 \}$.

$C = \mathbb{H}_\infty / \Gamma_\infty \subset \mathbb{H}^3/\Gamma$.

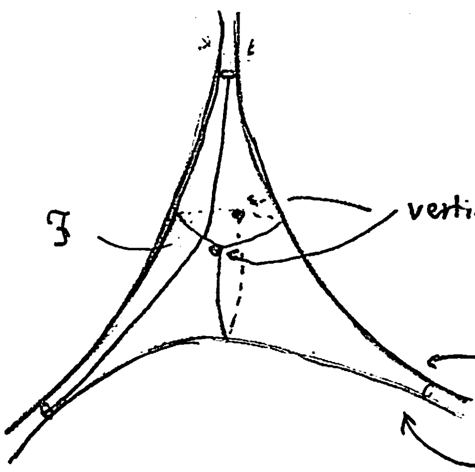


The Ford complex (cut locus) of M : $\mathfrak{F}$.

$$\mathfrak{F} := \{ x \in M \mid \text{there are two shortest geodesics to } C \}.$$



a 2 dim'l example



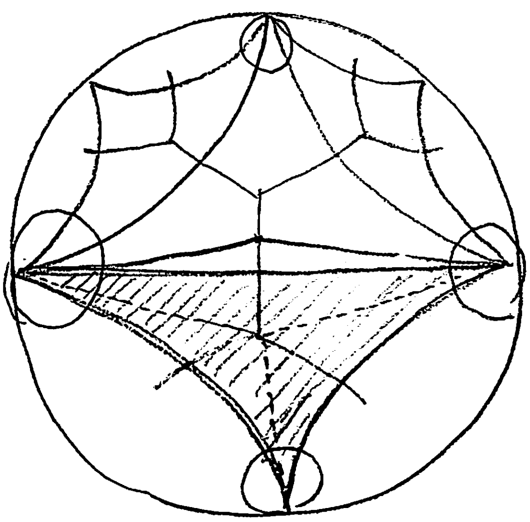
The canonical decomposition of M = an ideal polyhedral triangulation of M dual to $\mathfrak{F}$

e.g. $x \in \mathfrak{F}$

γ_1, γ_2 : shortest geodesics

$\gamma_1 \cup \gamma_2 \sim \gamma$: geodesic $\leftarrow$ an edge.

$\tilde{\mathcal{F}}$



$\tilde{\mathcal{D}}$

cf. J. Weeks — canonical decomposition

Epstein - Penner — Euclidean decomposition of

Minkowski model : convex hull construction for cusped hyperbolic manifold of finite

Akiyoshi - Sakuma — Comparing two convex hull constructions
EPH decomposition.

Kojima — totally geodesic bdry case.

Penner, Bowditch - Epstein — Cell-decomposition of Teichmüller space.

Ushijima.

Application to Knot theory.

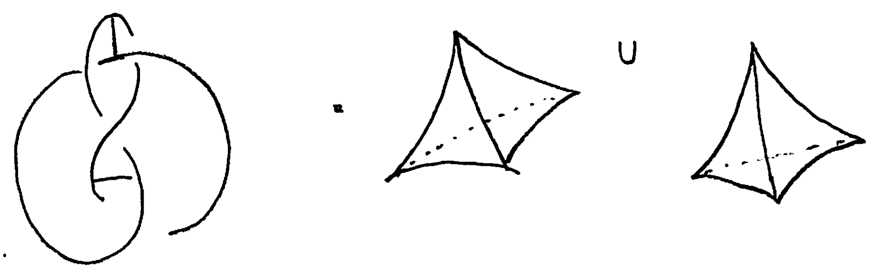
K, K' hyperbolic knots $\qquad K \cong K' \Leftrightarrow S^3 \setminus K \cong S^3 \setminus K'$

$\Leftrightarrow$ Mostow rigidity $\qquad \cong$ isometric

$\Leftrightarrow \quad \mathcal{D}_K \cong \mathcal{D}_{K'}$ combinatorially isomorphic.

$$\begin{aligned} Sym(S^3, K) &:= \pi_0 \, Diff(S^3, K) \\ &= \pi_0 \, Diff(S^3 - K) \\ &= Isom(S^3 - K) \\ &= Aut(\mathcal{D}_K) \end{aligned}$$

e.g.



Question $\quad$ 2-bridge knot complement
Find canonical decomposition.

Hint $\quad$ Jorgensen decomposition of punctured torus bundle over S^1.

Sakuma - Weeks
2 bridge knot $K(r)$ of slope r.
a topological ideal triangulation of $S^3 \setminus K(r)$, a candidate of the canonical decomposition.

Akiyoshi - Sakuma - Wada - Yamashita. $\qquad$ Affirmative!

LNM: Punctured torus groups and 2-bridge knot groups (I).

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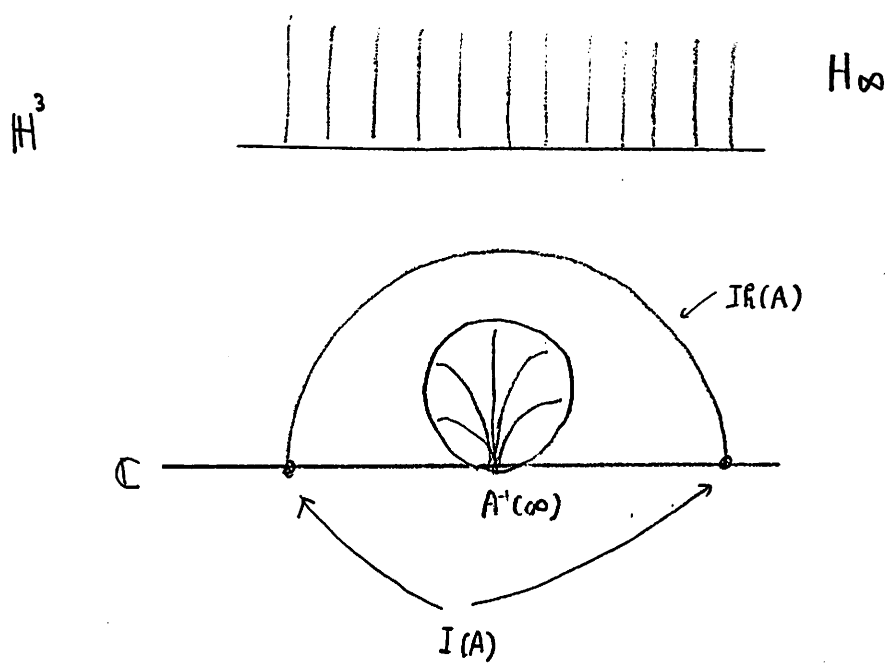
[Futer - Gueritaud]. Sakuma - Weeks decomposition is geometric
(realized by totally geodesic triangles).

[Gueritaud]. S-W decomposition is canonical.
EPH - conjecture is proved.
(by Sakuma-Akiyoshi)

Jorgensen's Idea.

$A \in PSL(2,\mathbb{C}) = \mathrm{Isom}^+ \mathbb{H}^3$. $\quad A(\infty) \neq \infty$.

The isometric hemisphere $Ih(A)$
circle $I(A)$.



$Ih(A) := \{ x \in \mathbb{H}^3 \mid d(x, H_\infty) = d(x, A^{-1}(H_\infty)) \}$

$I(A) = \partial Ih(A) \subset \mathbb{C}$.

Prop $\quad I(A) = \{ z \in A \mid |A'(z)| = 1 \}$ $\quad$ i.e. dA_z is a Euclidean isometry.

$\quad = \{ z \in A \mid |cz+d| = 1 \}$.

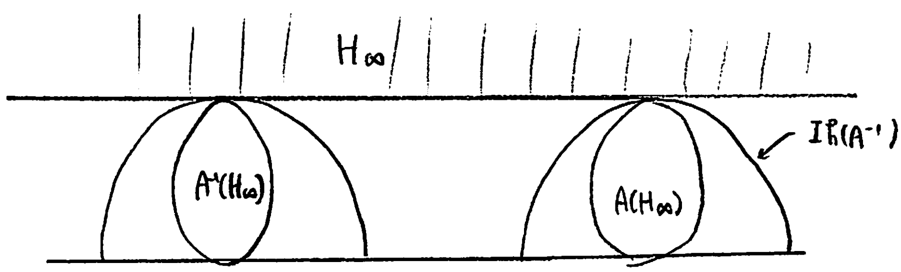
radius $= \dfrac{1}{|c|}$

centered at $-\dfrac{d}{c}$.

$Eh(A)$ = the "exterior" of $Ih(A)$ $\supset H_\infty$

$Dh(A)$ = the "interior" "

$A(Ih(A)) = Ih(A^{-1})$. $\qquad A(Eh(A)) = Dh(A)$



Ford domain of a Kleinian group. Γ.

$$Ph(\Gamma) := \bigcap_{A \in \Gamma - \Gamma_\infty} Eh(A)$$

Theorem. (1) $Ph(\Gamma)$ is a fundamental domain of Γ mod Γ_∞

(2) $M = \mathbb{H}^3/\Gamma$ has a unique cusp $\Rightarrow$ $\mathfrak{F} = p(\partial Ph(\Gamma))$

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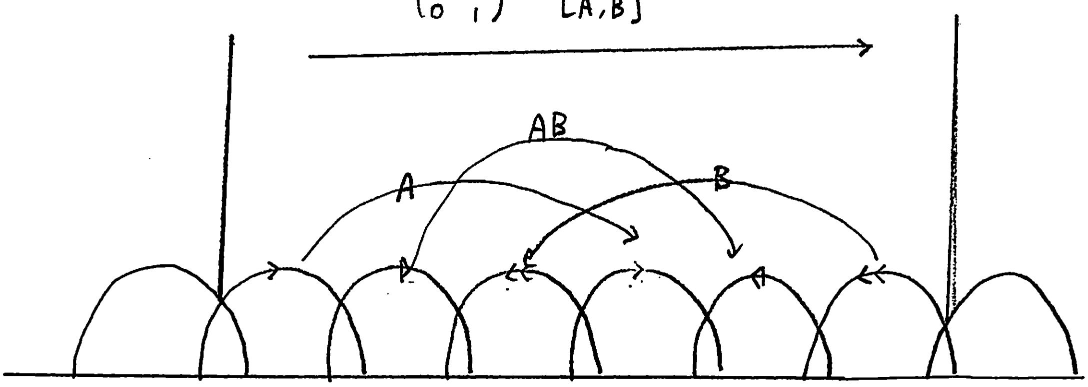
<u>Ex</u>. Hyperbolic punctured torus with $\mathbb{Z}/3\mathbb{Z}$ symmetry.

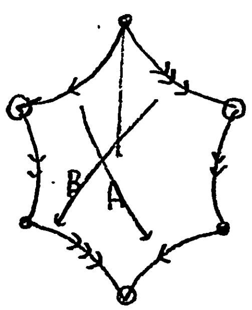
$$\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} = [A, B]$$



$\Gamma = \langle A, B \rangle \subset \mathrm{Isom}^+ \mathbb{H}^2$

<u>Fact</u> the Exterior above is a Ford domain

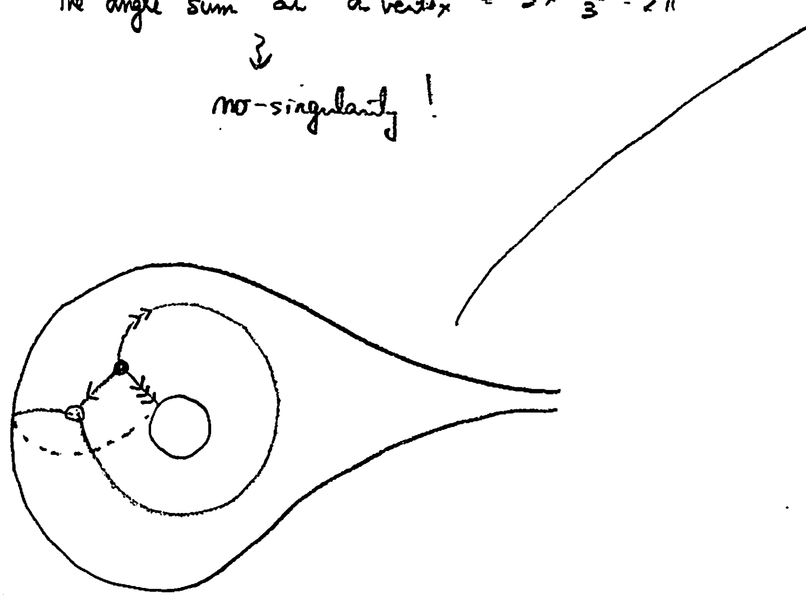
the fundamental domain $=$ $P\bar{h}(\Gamma) \cap$ (the fundamental domain of $[A, B]$)



the angle sum at a vertex $= 3 \times \frac{2\pi}{3} = 2\pi$

$\rightsquigarrow$ no-singularity !

$\searrow p$



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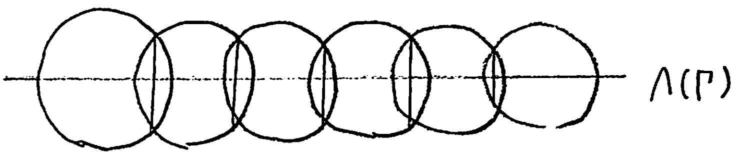
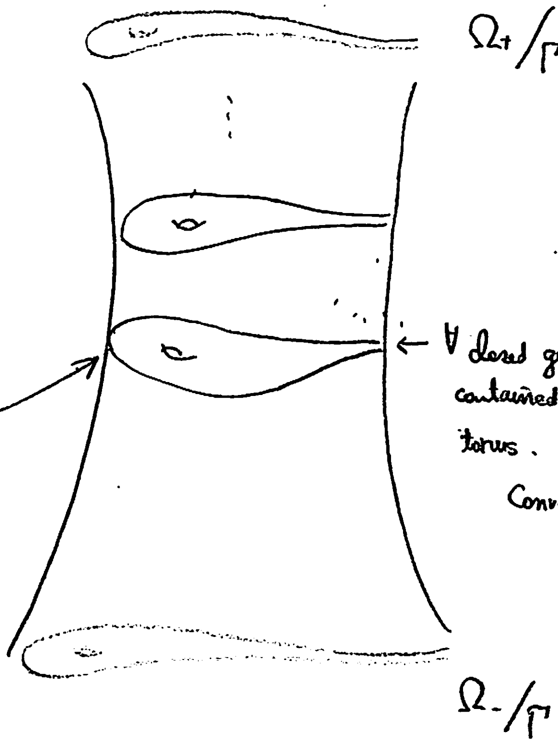
$\Gamma \subset \mathrm{Isom}^+(\mathbb{H}^3)$.



Bears : $\mathbb{H}^3 \cup \Omega / \Gamma$ is determined by $\Omega_\pm / \Gamma$.

has a conformal str.

From the above



$\Lambda(\Gamma) = \mathbb{R} \cup \{\infty\}$

$\Omega(\Gamma) = \hat{\mathbb{C}} \setminus \hat{\mathbb{R}} = \Omega_+ \cup \Omega_-$

$\forall$ closed geodesic is contained in the punctured torus.

Convex core

$\mathbb{H}^3 \dot{\cup} \Omega(\Gamma) / \Gamma \cong T \times [-1, 1]$

top. $\uparrow$

a punctured torus

$\mathbb{H}^3 / \Gamma \cong T \times (-1, 1)$

Deformations of Γ. (Jorgensen).

$\mathcal{F} = \{ \rho : \pi_1(T) \to PSL(2,\mathbb{R}) \mid \text{Fuchsian} \} / \text{conj} = \mathrm{Teich}(T) \cong \mathbb{H}^2$

$\cap$

$Q\mathcal{F} = \{ \quad " \quad PSL(2,\mathbb{C}) \mid \text{QuasiFuchsian} \} / \text{conj} \cong \mathrm{Teich}(T) \times \mathrm{Teich}(T) \cong \mathbb{H}^2 \times \mathbb{H}^2$

$\cap$

$\overline{Q\mathcal{F}} = D := \{ \rho \mid \rho : \text{discrete faithful} \}$

cf. Minsky
HP

Thurston's ending lamination theorem

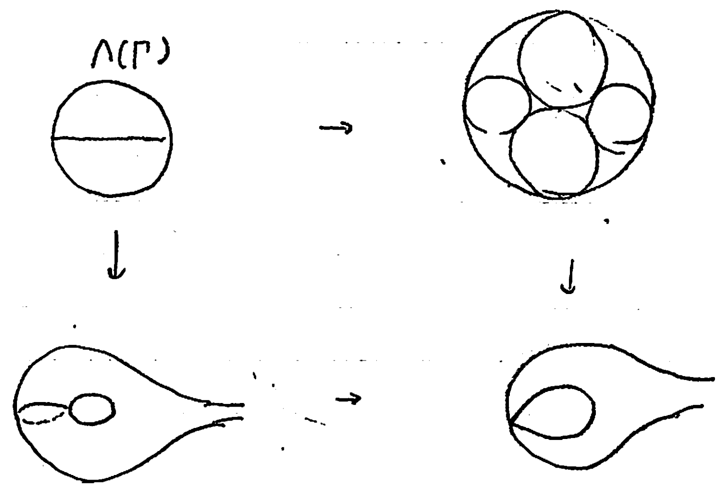
$$\begin{array}{ccc} \mathbb{H}^2\times\mathbb{H}^2 - \mathrm{diag}(\partial\mathbb{H}^2) & \longrightarrow & D \\ \rotatebox{90}{$\in$} & & \rotatebox{90}{$\in$} \\ \text{ending lamination} & \longmapsto & \rho \end{array}$$

ends

(1) $\Omega_+ \approx \mathbb{H}^2$

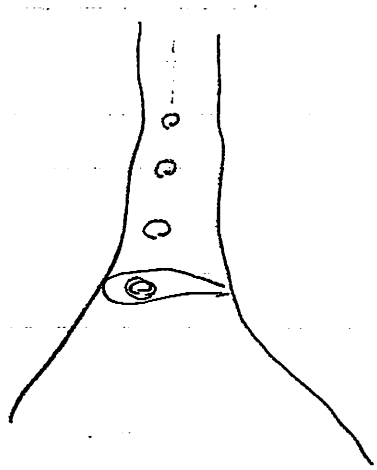
(2) $\Omega_+ = \amalg$ dist.

$\Lambda(\Gamma)$



(3) $\Omega_+ = \phi$.

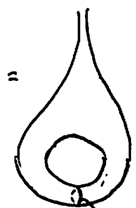
Convex core $\supset T\times[0,\infty)$



$$\nu_+(\rho) \in \overline{\mathbb{H}^2} = \overline{\mathrm{Teich}(T)} = \mathrm{Teich}(T) \cup \mathcal{PML} = \mathbb{H}^2 \cup \partial \mathbb{H}^2 = \mathbb{H}^2 \cup \hat{\mathbb{R}}$$

Case 1. $\Omega_+ \cong \mathbb{H}^2 \Rightarrow \nu_+(\rho) = [\Omega_+ / \rho(\pi_1(T))] \in \mathrm{Teich}(T) = \mathbb{H}^2$

case 2 $\Omega_+ \cong \amalg$ disk. $= \nu_+(\rho) = [\alpha_r] \longleftrightarrow r \in \hat{\mathbb{Q}}$

$\Omega_+ / \rho =$ 

$\rho(\alpha_r)$: parabolic

Case 3 $\Omega_+ = \emptyset$

$\nu_+(\rho) = \nu_\infty$

Example. $S^3 -$  $= M = T \times \mathbb{R} / (x,t) \sim (\varphi(x), t+1)$.

$$\varphi = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix} : T \to T$$

$$\tilde{M} := T \times \mathbb{R} \to M$$

$$\begin{array}{ccc} \pi_1(T) = \pi_1(\tilde{M}) & \xrightarrow{\rho \text{ discrete}} & \\ \downarrow & & \\ \pi_1(M) & \longrightarrow & PSL(2,\mathbb{C}) \end{array}$$

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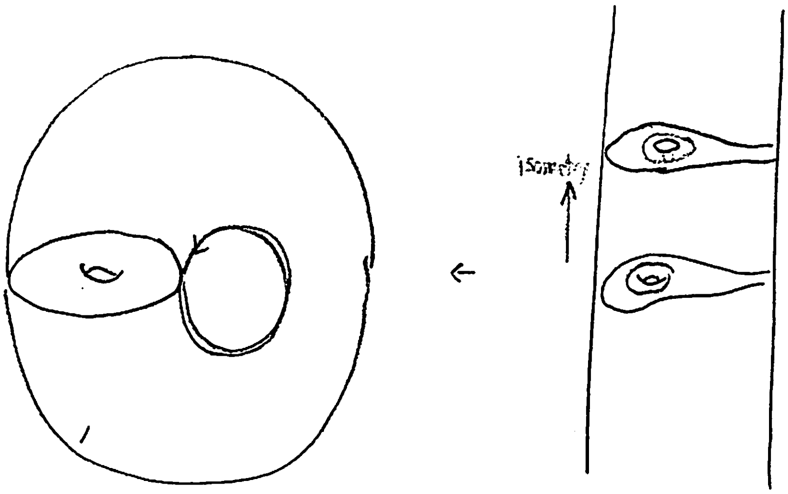
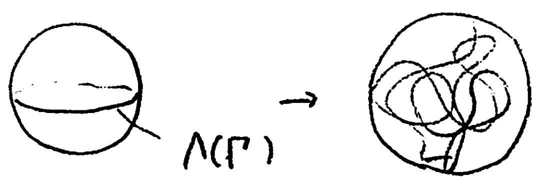


Eigenvalues : $\lambda_+ > 1 > \lambda_- > 0$

ν_+ - the slope of the eigenvector of λ_+

ν_- = λ_-

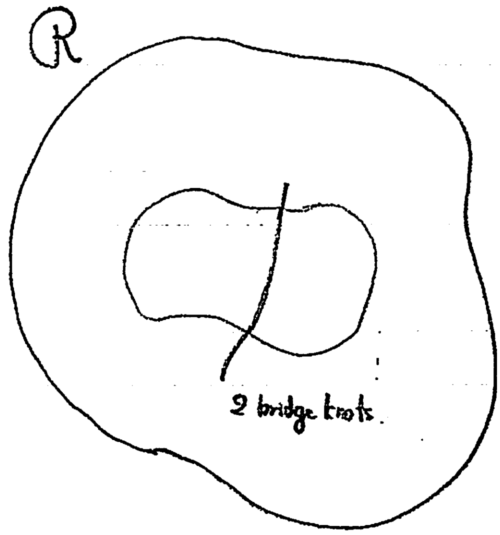
Cannon - Thurston map .



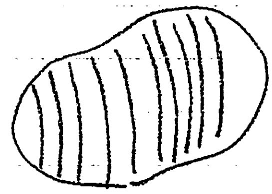
$$\overline{QF} \xrightarrow{\text{bij}} \mathbb{H}^2 \times \mathbb{H}^2 - \mathrm{diag}(\partial \mathbb{H}^2)$$

$$\cap$$

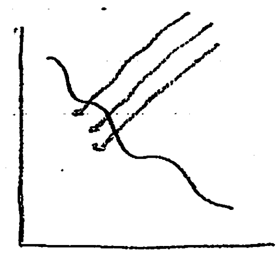
$$\mathcal{R} = \{ \rho : \pi_1(T) \to PSL(2,\mathbb{C}) \mid \text{type-pres} \} = \{ (x,y,z) \mid x^2+y^2+z^2 = xyz \} \Big/ \begin{matrix}(x,y,z)\\ \sim(x,-y,-z).\end{matrix}$$



P_{ν_+,ν_-}



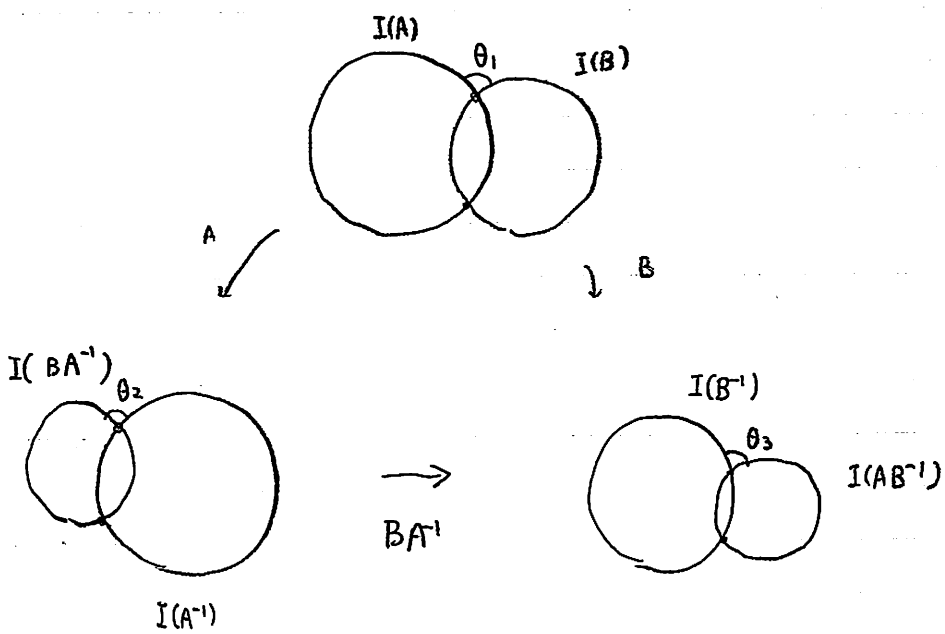
$\nu_+/\nu_- \in \mathbb{Q}$



Jorgensen Theory

1) Poincaré Theorem

2) A chain rule of isometric circles



$\theta_1 + \theta_2 + \theta_3 = 2\pi.$

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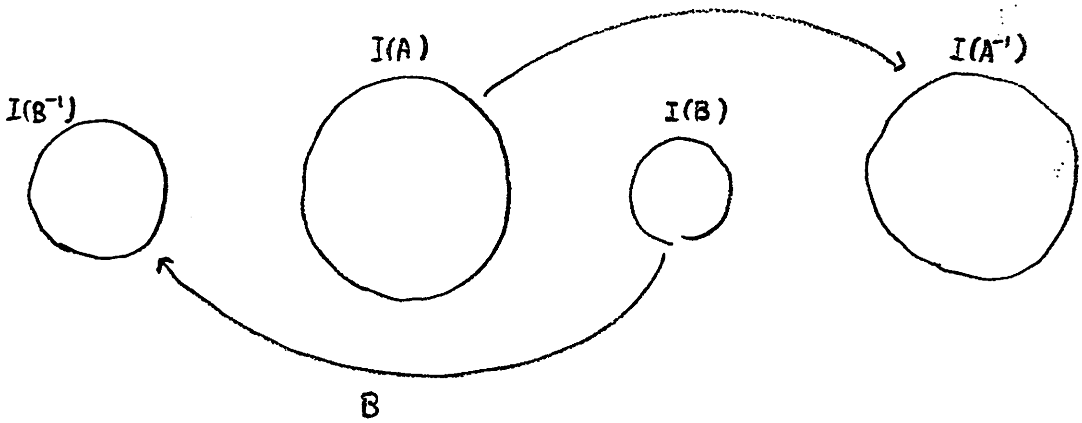
Chain rule cf. RIMS Kokyuroku 967 (1996) 58-70.

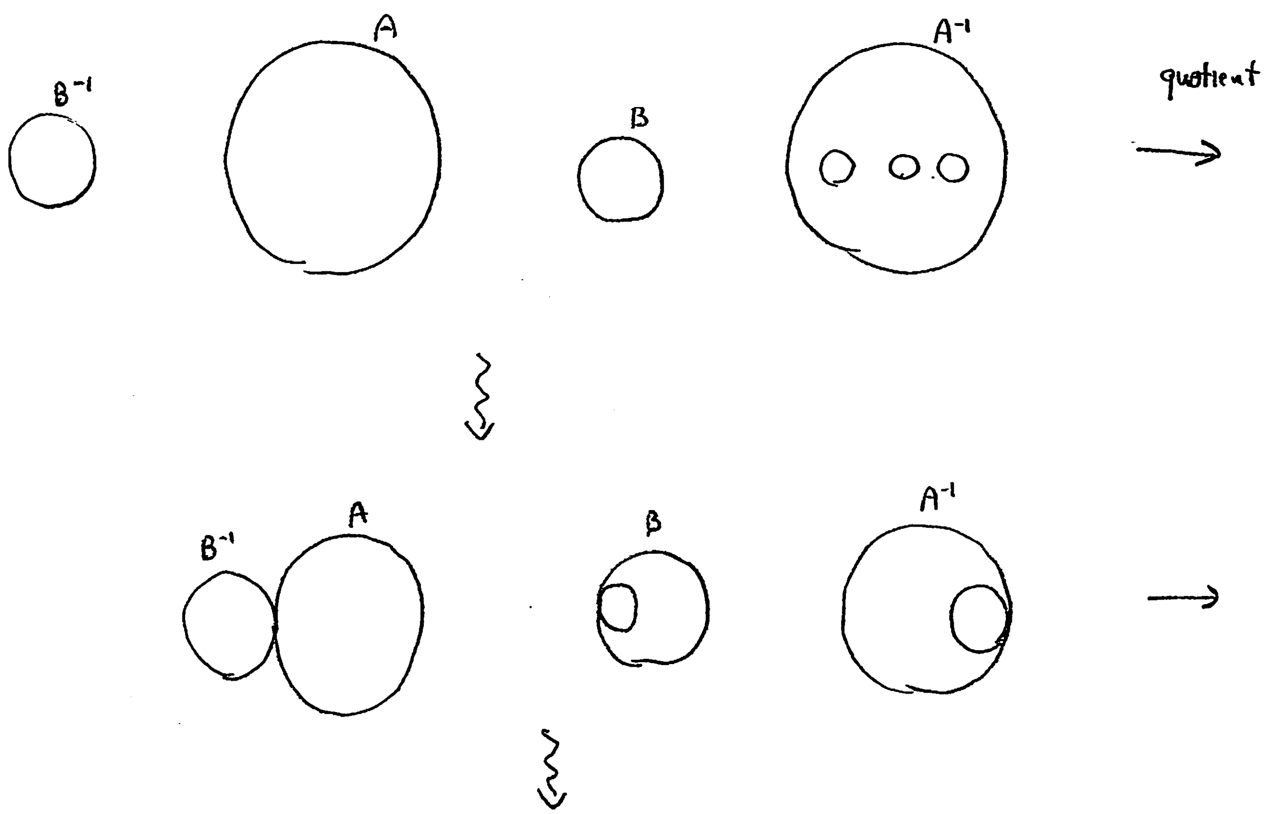
Schottky group.



$$E := \bigcap_{X \in \{A^{\pm 1}, B^{\pm 1}\}} E(X)$$

$\Gamma = \langle A, B \rangle$ free Kleinian group

$E \curvearrowright \Omega(\Gamma)$ has a fundamental domain E



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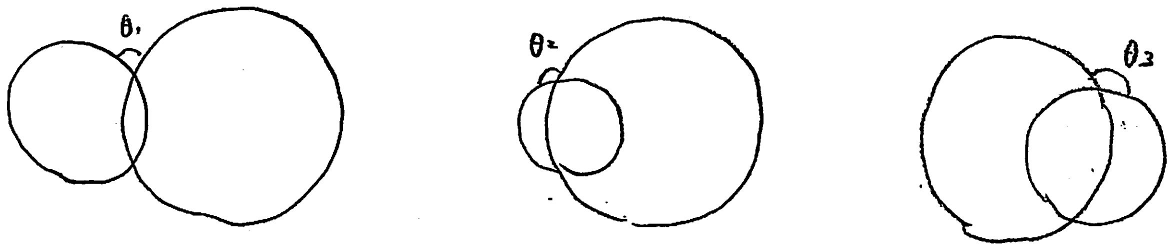
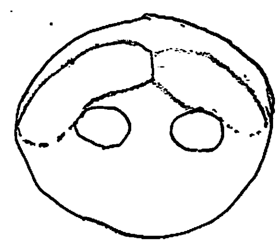
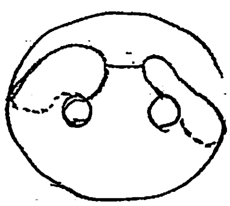
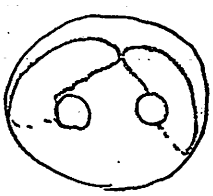
fundamental domain of $\langle A, B \rangle$.









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Theorem (Thurston)

M_φ : hyp $\Leftrightarrow$ φ pseudo Anosov.

genus 1 fibred knot.

$\begin{pmatrix} 1 & -1 \\ 1 & 0 \end{pmatrix}$

periodic.

$\begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$

p-Anosov